15 | ALTERNATING-CURRENT CIRCUITS

Figure 15.1 The current we draw into our houses is an alternating current (ac). Power lines transmit ac to our neighborhoods, where local power stations and transformers distribute it to our homes. In this chapter, we discuss how a transformer works and how it allows us to transmit power at very high voltages and minimal heating losses across the lines.

Chapter Outline

15.1 AC Sources
15.2 Simple AC Circuits
15.3 RLC Series Circuits with AC
15.4 Power in an AC Circuit
15.5 Resonance in an AC Circuit
15.6 Transformers

Introduction

Electric power is delivered to our homes by alternating current (ac) through high-voltage transmission lines. As explained in **Transformers**, transformers can then change the amplitude of the alternating potential difference to a more useful form. This lets us transmit power at very high voltages, minimizing resistive heating losses in the lines, and then furnish that power to homes at lower, safer voltages. Because constant potential differences are unaffected by transformers, this capability is more difficult to achieve with direct-current transmission.

In this chapter, we use Kirchhoff's laws to analyze four simple circuits in which ac flows. We have discussed the use of the resistor, capacitor, and inductor in circuits with batteries. These components are also part of ac circuits. However, because ac is required, the constant source of emf supplied by a battery is replaced by an ac voltage source, which produces an oscillating emf.

15.1 | AC Sources

Learning Objectives

By the end of the section, you will be able to:

- Explain the differences between direct current (dc) and alternating current (ac)
- Define characteristic features of alternating current and voltage, such as the amplitude or peak and the frequency

Most examples dealt with so far in this book, particularly those using batteries, have constant-voltage sources. Thus, once the current is established, it is constant. **Direct current (dc)** is the flow of electric charge in only one direction. It is the steady state of a constant-voltage circuit.

Most well-known applications, however, use a time-varying voltage source. **Alternating current (ac)** is the flow of electric charge that periodically reverses direction. An ac is produced by an alternating emf, which is generated in a power plant, as described in **Induced Electric Fields**. If the ac source varies periodically, particularly sinusoidally, the circuit is known as an ac circuit. Examples include the commercial and residential power that serves so many of our needs.

The ac voltages and frequencies commonly used in businesses and homes vary around the world. In a typical house, the potential difference between the two sides of an electrical outlet alternates sinusoidally with a frequency of 60 or 50 Hz and an amplitude of 156 or 311 V, depending on whether you live in the United States or Europe, respectively. Most people know the potential difference for electrical outlets is 120 V or 220 V in the US or Europe, but as explained later in the chapter, these voltages are not the peak values given here but rather are related to the common voltages we see in our electrical outlets. **Figure 15.2** shows graphs of voltage and current versus time for typical dc and ac power in the United States.

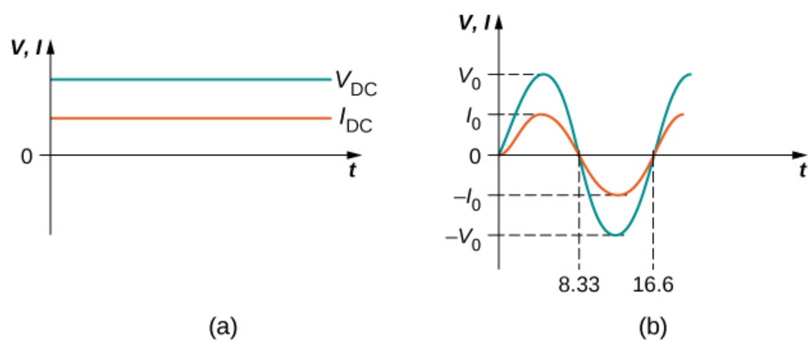


Figure 15.2 (a) The dc voltage and current are constant in time, once the current is established. (b) The voltage and current versus time are quite different for ac power. In this example, which shows 60-Hz ac power and time *t* in seconds, voltage and current are sinusoidal and are in phase for a simple resistance circuit. The frequencies and peak voltages of ac sources differ greatly.

Suppose we hook up a resistor to an ac voltage source and determine how the voltage and current vary in time across the resistor. **Figure 15.3** shows a schematic of a simple circuit with an ac voltage source. The voltage fluctuates sinusoidally with time at a fixed frequency, as shown, on either the battery terminals or the resistor. Therefore, the **ac voltage**, or the "voltage at a plug," can be given by

$$v = V_0 \sin \omega t, \tag{15.1}$$

where v is the voltage at time t, V_0 is the peak voltage, and ω is the angular frequency in radians per second. For a typical house in the United States, $V_0 = 156\text{ V}$ and $\omega = 120\pi\text{ rad/s}$, whereas in Europe, $V_0 = 311\text{ V}$ and $\omega = 100\pi\text{ rad/s}$.

For this simple resistance circuit, $I = V/R$, so the **ac current**, meaning the current that fluctuates sinusoidally with time at a fixed frequency, is

$$i = I_0 \sin \omega t, \quad (15.2)$$

where i is the current at time t and I_0 is the peak current and is equal to V_0/R. For this example, the voltage and current are said to be in phase, meaning that their sinusoidal functional forms have peaks, troughs, and nodes in the same place. They oscillate in sync with each other, as shown in **Figure 15.2**(b). In these equations, and throughout this chapter, we use lowercase letters (such as i) to indicate instantaneous values and capital letters (such as I) to indicate maximum, or peak, values.

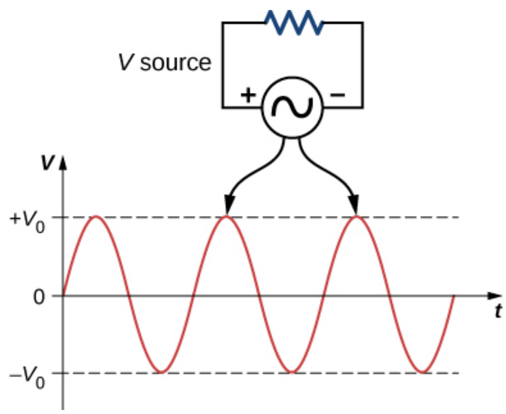


Figure 15.3 The potential difference V between the terminals of an ac voltage source fluctuates, so the source and the resistor have ac sine waves on top of each other. The mathematical expression for v is given by $v = V_0 \sin \omega t$.

Current in the resistor alternates back and forth just like the driving voltage, since $I = V/R$. If the resistor is a fluorescent light bulb, for example, it brightens and dims 120 times per second as the current repeatedly goes through zero. A 120-Hz flicker is too rapid for your eyes to detect, but if you wave your hand back and forth between your face and a fluorescent light, you will see the stroboscopic effect of ac.

15.1 Check Your Understanding If a European ac voltage source is considered, what is the time difference between the zero crossings on an ac voltage-versus-time graph?

15.2 | Simple AC Circuits

Learning Objectives

By the end of the section, you will be able to:

- Interpret phasor diagrams and apply them to ac circuits with resistors, capacitors, and inductors
- Define the reactance for a resistor, capacitor, and inductor to help understand how current in the circuit behaves compared to each of these devices

In this section, we study simple models of ac voltage sources connected to three circuit components: (1) a resistor, (2) a capacitor, and (3) an inductor. The power furnished by an ac voltage source has an emf given by

$$v(t) = V_0 \sin \omega t,$$

as shown in **Figure 15.4**. This sine function assumes we start recording the voltage when it is $v = 0\text{ V}$ at a time of $t = 0\text{ s}$. A phase constant may be involved that shifts the function when we start measuring voltages, similar to the phase constant in the waves we studied in **Waves (http://cnx.org/content/m58367/latest/)** . However, because we are free to choose when we start examining the voltage, we can ignore this phase constant for now. We can measure this voltage across the circuit components using one of two methods: (1) a quantitative approach based on our knowledge of circuits, or (2) a graphical approach that is explained in the coming sections.

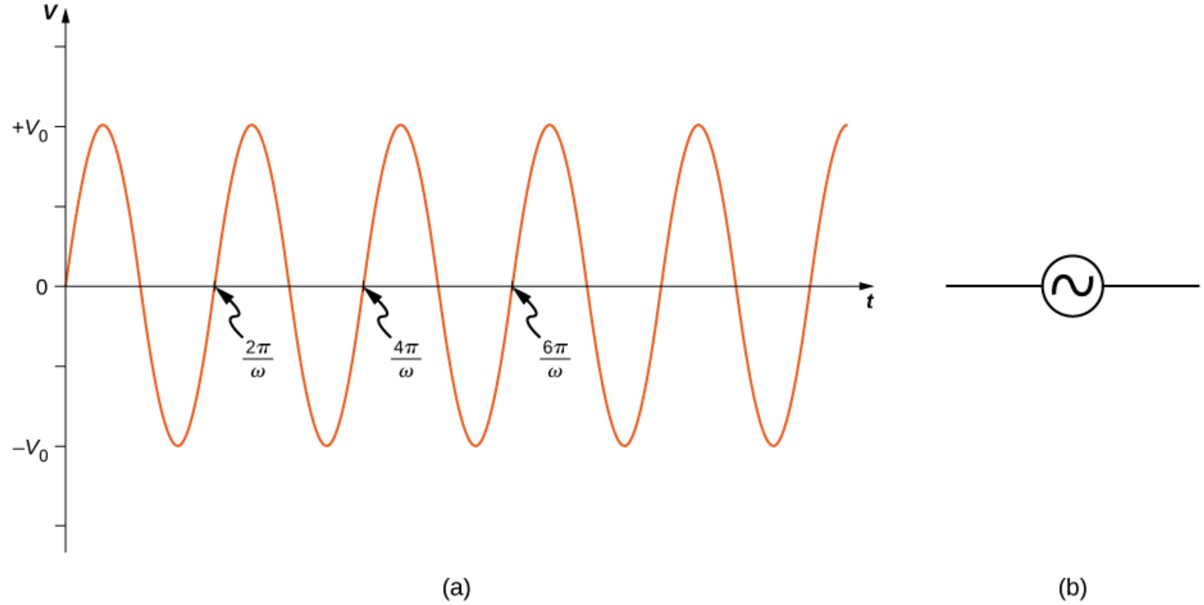


Figure 15.4 (a) The output $v(t) = V_0 \sin \omega t$ of an ac generator. (b) Symbol used to represent an ac voltage source in a circuit diagram.

Resistor

First, consider a resistor connected across an ac voltage source. From Kirchhoff's loop rule, the instantaneous voltage across the resistor of **Figure 15.5**(a) is

$$v_R(t) = V_0 \sin \omega t$$

and the instantaneous current through the resistor is

$$i_R(t) = \frac{v_R(t)}{R} = \frac{V_0}{R} \sin \omega t = I_0 \sin \omega t.$$

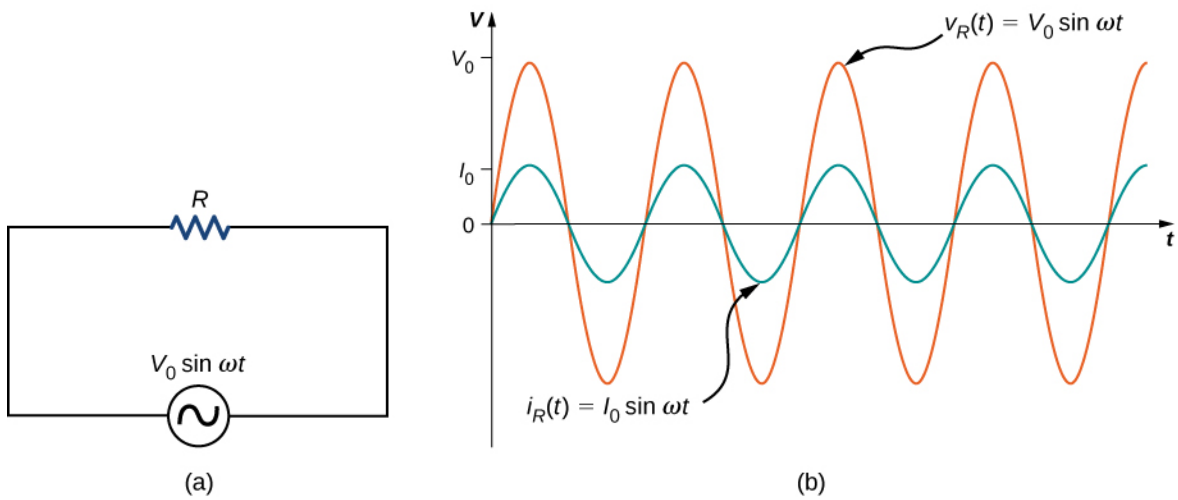


Figure 15.5 (a) A resistor connected across an ac voltage source. (b) The current $i_R(t)$ through the resistor and the voltage $v_R(t)$ across the resistor. The two quantities are in phase.

Here, $I_0 = V_0/R$ is the amplitude of the time-varying current. Plots of $i_R(t)$ and $v_R(t)$ are shown in **Figure 15.5**(b). Both curves reach their maxima and minima at the same times, that is, the current through and the voltage across the resistor are in phase.

Graphical representations of the phase relationships between current and voltage are often useful in the analysis of ac circuits. Such representations are called *phasor diagrams*. The phasor diagram for $i_R(t)$ is shown in **Figure 15.6**(a), with the current on the vertical axis. The arrow (or phasor) is rotating counterclockwise at a constant angular frequency ω, so we are viewing it at one instant in time. If the length of the arrow corresponds to the current amplitude I_0, the projection of the rotating arrow onto the vertical axis is $i_R(t) = I_0 \sin \omega t$, which is the instantaneous current.

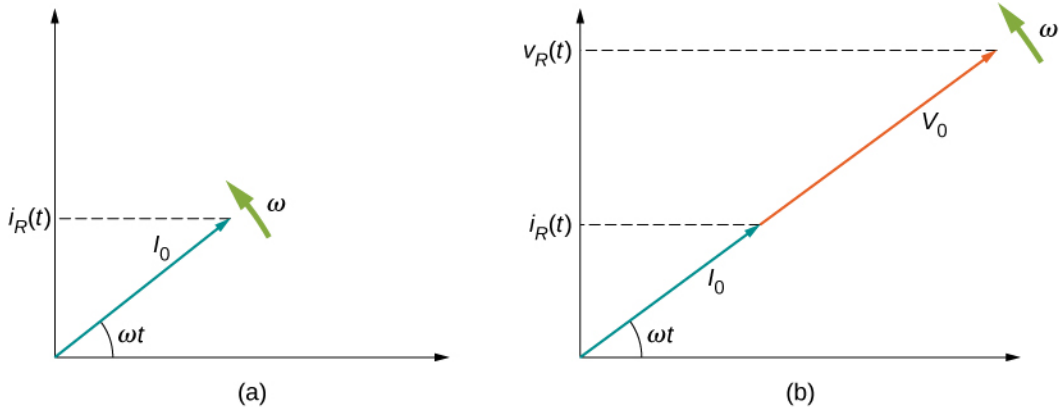


Figure 15.6 (a) The phasor diagram representing the current through the resistor of **Figure 15.5**. (b) The phasor diagram representing both $i_R(t)$ and $v_R(t)$.

The vertical axis on a phasor diagram could be either the voltage or the current, depending on the phasor that is being examined. In addition, several quantities can be depicted on the same phasor diagram. For example, both the current $i_R(t)$ and the voltage $v_R(t)$ are shown in the diagram of **Figure 15.6**(b). Since they have the same frequency and are in phase, their phasors point in the same direction and rotate together. The relative lengths of the two phasors are arbitrary because they represent different quantities; however, the ratio of the lengths of the two phasors can be represented by the resistance, since one is a voltage phasor and the other is a current phasor.

Capacitor

Now let's consider a capacitor connected across an ac voltage source. From Kirchhoff's loop rule, the instantaneous voltage across the capacitor of **Figure 15.7**(a) is

$$v_C(t) = V_0 \sin \omega t.$$

Recall that the charge in a capacitor is given by $Q = CV$. This is true at any time measured in the ac cycle of voltage. Consequently, the instantaneous charge on the capacitor is

$$q(t) = Cv_C(t) = CV_0 \sin \omega t.$$

Since the current in the circuit is the rate at which charge enters (or leaves) the capacitor,

$$i_C(t) = \frac{dq(t)}{dt} = \omega CV_0 \cos \omega t = I_0 \cos \omega t,$$

where $I_0 = \omega CV_0$ is the current amplitude. Using the trigonometric relationship $\cos \omega t = \sin(\omega t + \pi/2)$, we may express the instantaneous current as

$$i_C(t) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right).$$

Dividing V_0 by I_0, we obtain an equation that looks similar to Ohm's law:

$$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C. \tag{15.3}$$

The quantity X_C is analogous to resistance in a dc circuit in the sense that both quantities are a ratio of a voltage to a current. As a result, they have the same unit, the ohm. Keep in mind, however, that a capacitor stores and discharges electric energy, whereas a resistor dissipates it. The quantity X_C is known as the **capacitive reactance** of the capacitor, or the opposition of a capacitor to a change in current. It depends inversely on the frequency of the ac source—high frequency leads to low capacitive reactance.

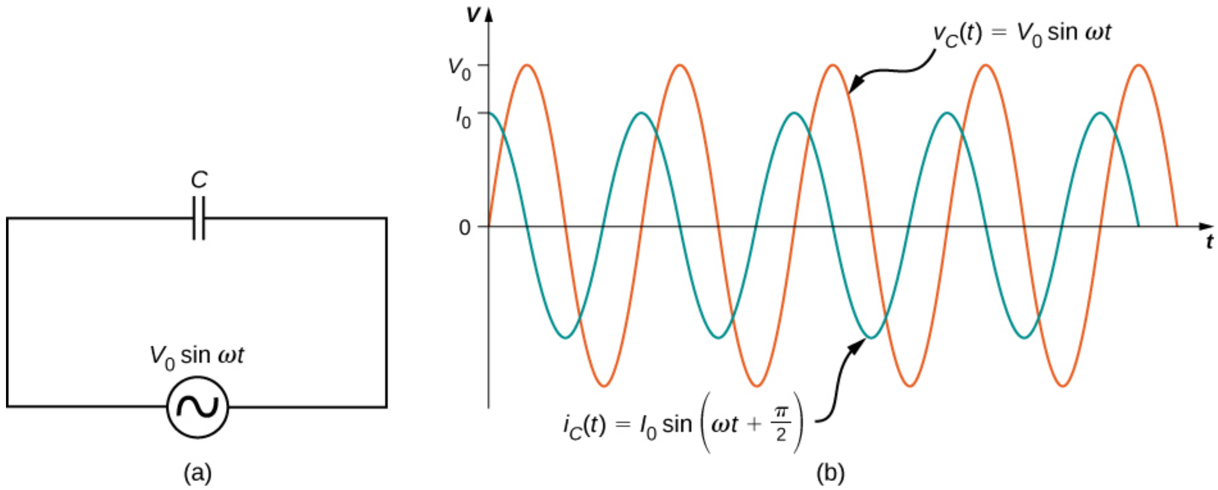


Figure 15.7 (a) A capacitor connected across an ac generator. (b) The current $i_C(t)$ through the capacitor and the voltage $v_C(t)$ across the capacitor. Notice that $i_C(t)$ leads $v_C(t)$ by $\pi/2$ rad.

A comparison of the expressions for $v_C(t)$ and $i_C(t)$ shows that there is a phase difference of $\pi/2$ rad between them. When these two quantities are plotted together, the current peaks a quarter cycle (or $\pi/2$ rad) ahead of the voltage, as illustrated in **Figure 15.7**(b). The current through a capacitor leads the voltage across a capacitor by $\pi/2$ rad, or a quarter of a cycle.

The corresponding phasor diagram is shown in **Figure 15.8**. Here, the relationship between $i_C(t)$ and $v_C(t)$ is represented by having their phasors rotate at the same angular frequency, with the current phasor leading by $\pi/2$ rad.

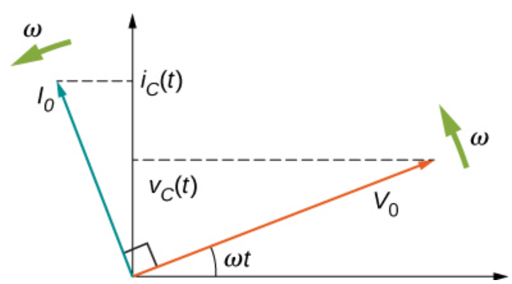


Figure 15.8 The phasor diagram for the capacitor of **Figure 15.7**. The current phasor leads the voltage phasor by $\pi/2$ rad as they both rotate with the same angular frequency.

To this point, we have exclusively been using peak values of the current or voltage in our discussion, namely, I_0 and V_0. However, if we average out the values of current or voltage, these values are zero. Therefore, we often use a second convention called the root mean square value, or rms value, in discussions of current and voltage. The rms operates in reverse of the terminology. First, you square the function, next, you take the mean, and then, you find the square root. As a result, the rms values of current and voltage are not zero. Appliances and devices are commonly quoted with rms values for their operations, rather than peak values. We indicate rms values with a subscript attached to a capital letter (such as I_{rms}).

Although a capacitor is basically an open circuit, an **rms current**, or the root mean square of the current, appears in a circuit with an ac voltage applied to a capacitor. Consider that

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}, \tag{15.4}$$

where I_0 is the peak current in an ac system. The **rms voltage**, or the root mean square of the voltage, is

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}, \tag{15.5}$$

where V_0 is the peak voltage in an ac system. The rms current appears because the voltage is continually reversing, charging, and discharging the capacitor. If the frequency goes to zero, which would be a dc voltage, X_C tends to infinity, and the current is zero once the capacitor is charged. At very high frequencies, the capacitor's reactance tends to zero—it has a negligible reactance and does not impede the current (it acts like a simple wire).

Inductor

Lastly, let's consider an inductor connected to an ac voltage source. From Kirchhoff's loop rule, the voltage across the inductor L of **Figure 15.9**(a) is

$$v_L(t) = V_0 \sin \omega t. \tag{15.6}$$

The emf across an inductor is equal to $\varepsilon = -L(di_L/dt)$; however, the potential difference across the inductor is $v_L(t) = Ldi_L(t)/dt$, because if we consider that the voltage around the loop must equal zero, the voltage gained from the ac source must dissipate through the inductor. Therefore, connecting this with the ac voltage source, we have

$$\frac{di_L(t)}{dt} = \frac{V_0}{L} \sin \omega t.$$

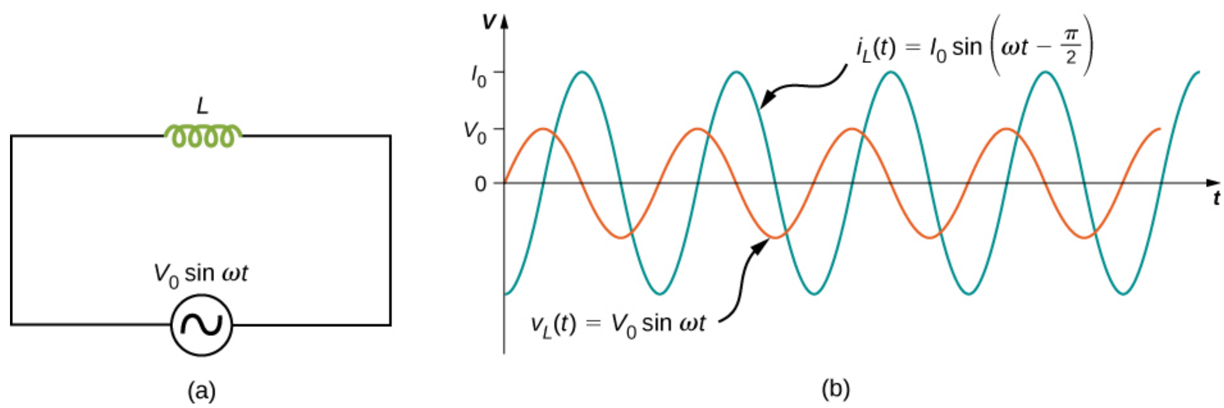


Figure 15.9 (a) An inductor connected across an ac generator. (b) The current $i_L(t)$ through the inductor and the voltage $v_L(t)$ across the inductor. Here $i_L(t)$ lags $v_L(t)$ by $\pi/2$ rad.

The current $i_L(t)$ is found by integrating this equation. Since the circuit does not contain a source of constant emf, there is no steady current in the circuit. Hence, we can set the constant of integration, which represents the steady current in the circuit, equal to zero, and we have

$$i_L(t) = -\frac{V_0}{\omega L}\cos \omega t = \frac{V_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) = I_0 \sin\left(\omega t - \frac{\pi}{2}\right), \tag{15.7}$$

where $I_0 = V_0/\omega L.$ The relationship between V_0 and I_0 may also be written in a form analogous to Ohm's law:

$$\frac{V_0}{I_0} = \omega L = X_L. \tag{15.8}$$

The quantity X_L is known as the **inductive reactance** of the inductor, or the opposition of an inductor to a change in current; its unit is also the ohm. Note that X_L varies directly as the frequency of the ac source—high frequency causes high inductive reactance.

A phase difference of $\pi/2$ rad occurs between the current through and the voltage across the inductor. From **Equation 15.6** and **Equation 15.7**, the current through an inductor lags the potential difference across an inductor by $\pi/2$ rad, or a quarter of a cycle. The phasor diagram for this case is shown in **Figure 15.10**.

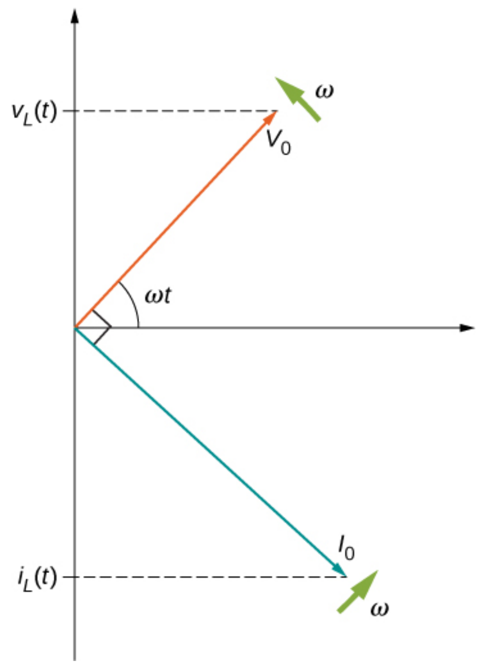


Figure 15.10 The phasor diagram for the inductor of **Figure 15.9**. The current phasor lags the voltage phasor by $\pi/2$ rad as they both rotate with the same angular frequency.

An animation from the University of New South Wales **AC Circuits (https://openstaxcollege.org/l/21accircuits)** illustrates some of the concepts we discuss in this chapter. They also include wave and phasor diagrams that evolve over time so that you can get a better picture of how each changes over time.

Example 15.1

Simple AC Circuits

An ac generator produces an emf of amplitude 10 V at a frequency $f = 60\,\text{Hz}$. Determine the voltages across and the currents through the circuit elements when the generator is connected to (a) a $100\text{-}\Omega$ resistor, (b) a $10\text{-}\mu\text{F}$ capacitor, and (c) a 15-mH inductor.

Strategy

The entire AC voltage across each device is the same as the source voltage. We can find the currents by finding the reactance X of each device and solving for the peak current using $I_0 = V_0/X$.

Solution

The voltage across the terminals of the source is

$$v(t) = V_0 \sin \omega t = (10\text{ V}) \sin 120\pi t,$$

where $\omega = 2\pi f = 120\pi\text{ rad/s}$ is the angular frequency. Since $v(t)$ is also the voltage across each of the elements, we have

$$v(t) = v_R(t) = v_C(t) = v_L(t) = (10\text{ V}) \sin 120\pi t.$$

a. When $R = 100\,\Omega$, the amplitude of the current through the resistor is

$$I_0 = V_0/R = 10\text{ V}/100\,\Omega = 0.10\text{ A},$$

so

$$i_R(t) = (0.10\text{ A}) \sin 120\pi t.$$

b. From **Equation 15.3**, the capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{(120\pi\text{ rad/s})(10 \times 10^{-6}\text{F})} = 265\,\Omega,$$

so the maximum value of the current is

$$I_0 = \frac{V_0}{X_C} = \frac{10\text{ V}}{265\,\Omega} = 3.8 \times 10^{-2}\text{ A}$$

and the instantaneous current is given by

$$i_C(t) = (3.8 \times 10^{-2}\text{ A}) \sin\left(120\pi t + \frac{\pi}{2}\right).$$

c. From **Equation 15.8**, the inductive reactance is

$$X_L = \omega L = (120\pi\text{ rad/s})(15 \times 10^{-3}\text{ H}) = 5.7\,\Omega.$$

The maximum current is therefore

$$I_0 = \frac{10\text{ V}}{5.7\,\Omega} = 1.8\text{ A}$$

and the instantaneous current is

$$i_L(t) = (1.8\text{ A}) \sin\left(120\pi t - \frac{\pi}{2}\right).$$

Significance

Although the voltage across each device is the same, the peak current has different values, depending on the reactance. The reactance for each device depends on the values of resistance, capacitance, or inductance.

15.2 Check Your Understanding Repeat **Example 15.1** for an ac source of amplitude 20 V and frequency 100 Hz.

15.3 | RLC Series Circuits with AC

Learning Objectives

By the end of the section, you will be able to:

- Describe how the current varies in a resistor, a capacitor, and an inductor while in series with an ac power source
- Use phasors to understand the phase angle of a resistor, capacitor, and inductor ac circuit and to understand what that phase angle means
- Calculate the impedance of a circuit

The ac circuit shown in **Figure 15.11**, called an *RLC* series circuit, is a series combination of a resistor, capacitor, and inductor connected across an ac source. It produces an emf of

$$v(t) = V_0 \sin \omega t.$$

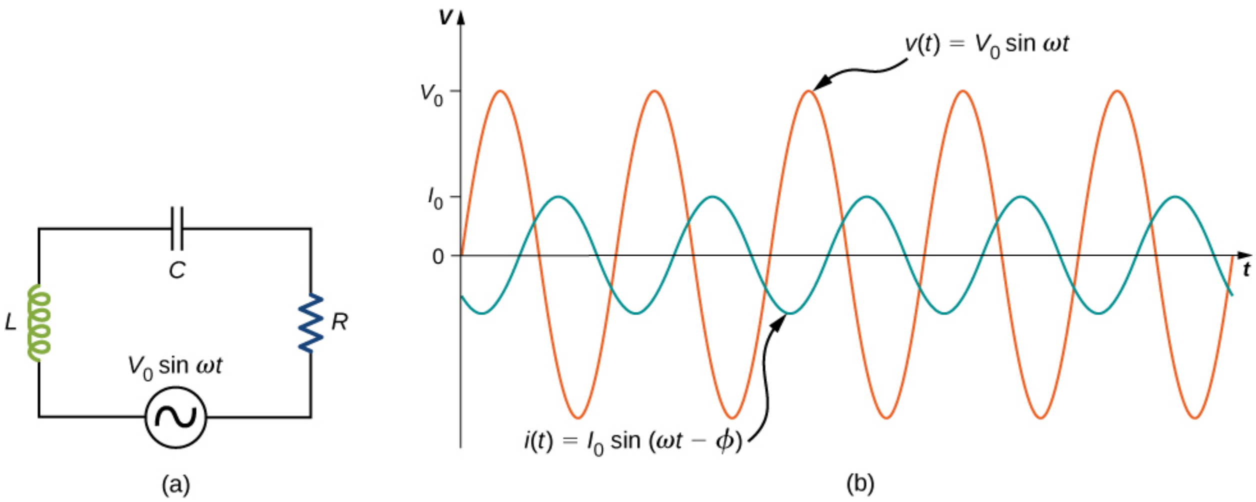


Figure 15.11 (a) An *RLC* series circuit. (b) A comparison of the generator output voltage and the current. The value of the phase difference ϕ depends on the values of *R*, *C*, and *L*.

Since the elements are in series, the same current flows through each element at all points in time. The relative phase between the current and the emf is not obvious when all three elements are present. Consequently, we represent the current by the general expression

$$i(t) = I_0 \sin(\omega t - \phi),$$

where I_0 is the current amplitude and ϕ is the **phase angle** between the current and the applied voltage. The phase angle is thus the amount by which the voltage and current are out of phase with each other in a circuit. Our task is to find I_0 and ϕ.

A phasor diagram involving $i(t)$, $v_R(t)$, $v_C(t)$, and $v_L(t)$ is helpful for analyzing the circuit. As shown in **Figure 15.12**, the phasor representing $v_R(t)$ points in the same direction as the phasor for $i(t)$; its amplitude is $V_R = I_0 R$. The $v_C(t)$ phasor lags the $i(t)$ phasor by $\pi/2$ rad and has the amplitude $V_C = I_0 X_C$. The phasor for $v_L(t)$ leads the $i(t)$ phasor by $\pi/2$ rad and has the amplitude $V_L = I_0 X_L$.

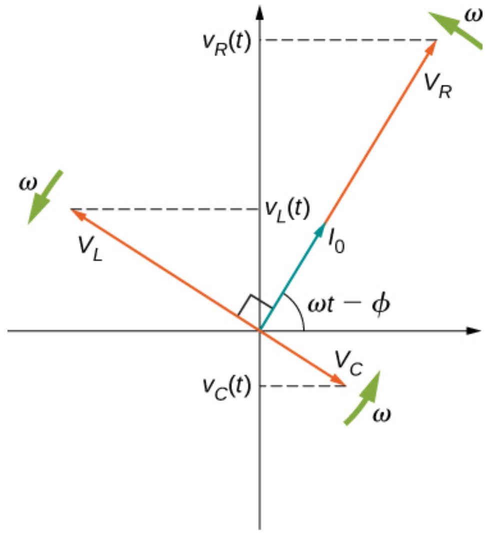


Figure 15.12 The phasor diagram for the *RLC* series circuit of **Figure 15.11**.

At any instant, the voltage across the *RLC* combination is $v_R(t) + v_L(t) + v_C(t) = v(t)$, the emf of the source. Since a component of a sum of vectors is the sum of the components of the individual vectors—for example, $(A + B)_y = A_y + B_y$ —the projection of the vector sum of phasors onto the vertical axis is the sum of the vertical projections of the individual phasors. Hence, if we add vectorially the phasors representing $v_R(t)$, $v_L(t)$, and $v_C(t)$ and then find the projection of the resultant onto the vertical axis, we obtain

$$v_R(t) + v_L(t) + v_C(t) = v(t) = V_0 \sin \omega t.$$

The vector sum of the phasors is shown in **Figure 15.13**. The resultant phasor has an amplitude V_0 and is directed at an angle ϕ with respect to the $v_R(t)$, or $i(t)$, phasor. The projection of this resultant phasor onto the vertical axis is $v(t) = V_0 \sin \omega t$. We can easily determine the unknown quantities I_0 and ϕ from the geometry of the phasor diagram. For the phase angle,

$$\phi = \tan^{-1} \frac{V_L - V_C}{V_R} = \tan^{-1} \frac{I_0 X_L - I_0 X_C}{I_0 R},$$

and after cancellation of I_0, this becomes

$$\phi = \tan^{-1} \frac{X_L - X_C}{R}. \tag{15.9}$$

Furthermore, from the Pythagorean theorem,

$$V_0 = \sqrt{V_R{}^2 + (V_L - V_C)^2} = \sqrt{(I_0 R)^2 + (I_0 X_L - I_0 X_C)^2} = I_0 \sqrt{R^2 + (X_L - X_C)^2}.$$

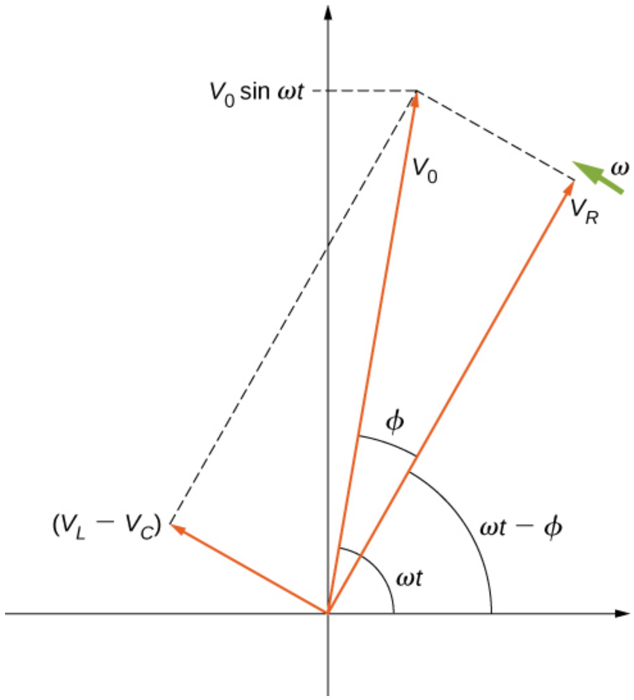


Figure 15.13 The resultant of the phasors for $v_L(t)$, $v_C(t)$, and $v_R(t)$ is equal to the phasor for $v(t) = V_0 \sin \omega t$. The $i(t)$ phasor (not shown) is aligned with the $v_R(t)$ phasor.

The current amplitude is therefore the ac version of Ohm's law:

$$I_0 = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_0}{Z}, \tag{15.10}$$

where

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{15.11}$$

is known as the **impedance** of the circuit. Its unit is the ohm, and it is the ac analog to resistance in a dc circuit, which measures the combined effect of resistance, capacitive reactance, and inductive reactance (**Figure 15.14**).

Figure 15.14 Power capacitors are used to balance the impedance of the effective inductance in transmission lines.

The *RLC* circuit is analogous to the wheel of a car driven over a corrugated road (**Figure 15.15**). The regularly spaced bumps in the road drive the wheel up and down; in the same way, a voltage source increases and decreases. The shock absorber acts like the resistance of the *RLC* circuit, damping and limiting the amplitude of the oscillation. Energy within the wheel system goes back and forth between kinetic and potential energy stored in the car spring, analogous to the shift between a maximum current, with energy stored in an inductor, and no current, with energy stored in the electric field of a capacitor. The amplitude of the wheel's motion is at a maximum if the bumps in the road are hit at the resonant frequency, which we describe in more detail in **Resonance in an AC Circuit**.

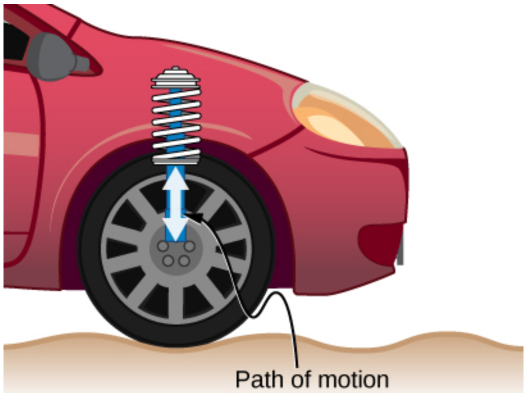


Figure 15.15 On a car, the shock absorber damps motion and dissipates energy. This is much like the resistance in an *RLC* circuit. The mass and spring determine the resonant frequency.

Problem-Solving Strategy: AC Circuits

To analyze an ac circuit containing resistors, capacitors, and inductors, it is helpful to think of each device's reactance and find the equivalent reactance using the rules we used for equivalent resistance in the past. Phasors are a great method to determine whether the emf of the circuit has positive or negative phase (namely, leads or lags other values). A mnemonic device of "ELI the ICE man" is sometimes used to remember that the emf (E) leads the current (I) in an inductor (L) and the current (I) leads the emf (E) in a capacitor (C).

Use the following steps to determine the emf of the circuit by phasors:

1. Draw the phasors for voltage across each device: resistor, capacitor, and inductor, including the phase angle in the circuit.
2. If there is both a capacitor and an inductor, find the net voltage from these two phasors, since they are antiparallel.
3. Find the equivalent phasor from the phasor in step 2 and the resistor's phasor using trigonometry or components of the phasors. The equivalent phasor found is the emf of the circuit.

Example 15.2

An *RLC* Series Circuit

The output of an ac generator connected to an *RLC* series combination has a frequency of 200 Hz and an amplitude of 0.100 V. If $R = 4.00\,\Omega$, $L = 3.00 \times 10^{-3}$ H, and $C = 8.00 \times 10^{-4}$ F, what are (a) the capacitive reactance, (b) the inductive reactance, (c) the impedance, (d) the current amplitude, and (e) the phase difference between the current and the emf of the generator?

Strategy

The reactances and impedance in (a)–(c) are found by substitutions into **Equation 15.3**, **Equation 15.8**, and **Equation 15.11**, respectively. The current amplitude is calculated from the peak voltage and the impedance. The phase difference between the current and the emf is calculated by the inverse tangent of the difference between the reactances divided by the resistance.

Solution

a. From **Equation 15.3**, the capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(200\text{ Hz})\left(8.00 \times 10^{-4}\text{ F}\right)} = 0.995\,\Omega.$$

b. From **Equation 15.8**, the inductive reactance is

$$X_L = \omega L = 2\pi(200\text{ Hz})\left(3.00 \times 10^{-3}\text{ H}\right) = 3.77\,\Omega.$$

c. Substituting the values of R, X_C, and X_L into **Equation 15.11**, we obtain for the impedance

$$Z = \sqrt{(4.00\,\Omega)^2 + (3.77\,\Omega - 0.995\,\Omega)^2} = 4.87\,\Omega.$$

d. The current amplitude is

$$I_0 = \frac{V_0}{Z} = \frac{0.100\text{ V}}{4.87\,\Omega} = 2.05 \times 10^{-2}\text{ A}.$$

e. From **Equation 15.9**, the phase difference between the current and the emf is

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{2.77\,\Omega}{4.00\,\Omega} = 0.607\text{ rad}.$$

Significance

The phase angle is positive because the reactance of the inductor is larger than the reactance of the capacitor.

15.3 Check Your Understanding Find the voltages across the resistor, the capacitor, and the inductor in the circuit of **Figure 15.11** using $v(t) = V_0 \sin \omega t$ as the output of the ac generator.

15.4 | Power in an AC Circuit

Learning Objectives

By the end of the section, you will be able to:

- Describe how average power from an ac circuit can be written in terms of peak current and voltage and of rms current and voltage
- Determine the relationship between the phase angle of the current and voltage and the average power, known as the power factor

A circuit element dissipates or produces power according to $P = IV$, where I is the current through the element and V is the voltage across it. Since the current and the voltage both depend on time in an ac circuit, the instantaneous power $p(t) = i(t)v(t)$ is also time dependent. A plot of $p(t)$ for various circuit elements is shown in **Figure 15.16**. For a resistor, $i(t)$ and $v(t)$ are in phase and therefore always have the same sign (see **Figure 15.5**). For a capacitor or inductor, the relative signs of $i(t)$ and $v(t)$ vary over a cycle due to their phase differences (see **Figure 15.7** and **Figure 15.9**). Consequently, $p(t)$ is positive at some times and negative at others, indicating that capacitive and inductive elements produce power at some instants and absorb it at others.

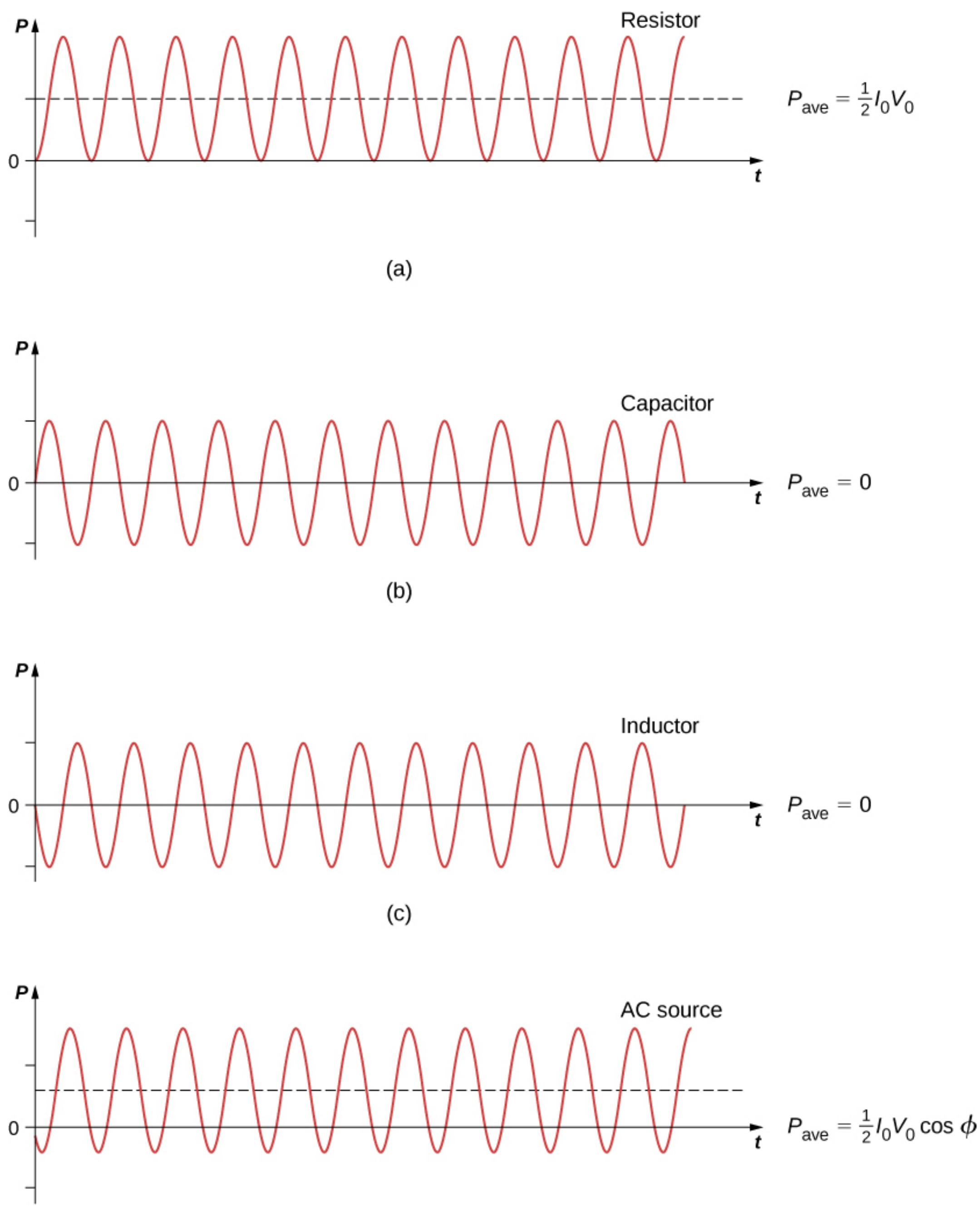


Figure 15.16 Graph of instantaneous power for various circuit elements. (a) For the resistor, $P_{\text{ave}} = I_0 V_0/2,$ whereas for (b) the capacitor and (c) the inductor, $P_{\text{ave}} = 0.$ (d) For the source, $P_{\text{ave}} = I_0 V_0(\cos \phi)/2,$ which may be positive, negative, or zero, depending on $\phi.$

Because instantaneous power varies in both magnitude and sign over a cycle, it seldom has any practical importance. What we're almost always concerned with is the power averaged over time, which we refer to as the **average power**. It is defined by the time average of the instantaneous power over one cycle:

$$P_{\text{ave}} = \frac{1}{T}\int_0^T p(t)dt,$$

where $T = 2\pi/\omega$ is the period of the oscillations. With the substitutions $v(t) = V_0 \sin \omega t$ and $i(t) = I_0 \sin (\omega t - \phi),$

this integral becomes

$$P_{\text{ave}} = \frac{I_0 V_0}{T}\int_0^T \sin(\omega t - \phi)\sin\omega t\, dt.$$

Using the trigonometric relation $\sin(A - B) = \sin A\cos B - \sin B\cos A,$ we obtain

$$P_{\text{ave}} = \frac{I_0 V_0 \cos\phi}{T}\int_0^T \sin\omega t dt - \frac{I_0 V_0 \sin\phi}{T}\int_0^T \sin^2\omega t\cos\omega t dt.$$

Evaluation of these two integrals yields

$$\frac{1}{T}\int_0^T \sin^2\omega t dt = \frac{1}{2}$$

and

$$\frac{1}{T}\int_0^T \sin\omega t\cos\omega t dt = 0.$$

Hence, the average power associated with a circuit element is given by

$$P_{\text{ave}} = \frac{1}{2}I_0 V_0 \cos\phi. \tag{15.12}$$

In engineering applications, $\cos\phi$ is known as the **power factor**, which is the amount by which the power delivered in the circuit is less than the theoretical maximum of the circuit due to voltage and current being out of phase. For a resistor, $\phi = 0,$ so the average power dissipated is

$$P_{\text{ave}} = \frac{1}{2}I_0 V_0.$$

A comparison of $p(t)$ and P_{ave} is shown in **Figure 15.16**(d). To make $P_{\text{ave}} = (1/2)I_0 V_0$ look like its dc counterpart, we use the rms values I_{rms} and V_{rms} of the current and the voltage. By definition, these are

$$I_{\text{rms}} = \sqrt{i^2_{\text{ave}}} \text{ and } V_{\text{rms}} = \sqrt{v^2_{\text{ave}}},$$

where

$$i^2_{\text{ave}} = \frac{1}{T}\int_0^T i^2(t)dt \text{ and } v^2_{\text{ave}} = \frac{1}{T}\int_0^T v^2(t)dt.$$

With $i(t) = I_0 \sin(\omega t - \phi)$ and $v(t) = V_0 \sin\omega t,$ we obtain

$$I_{\text{rms}} = \frac{1}{\sqrt{2}}I_0 \text{ and } V_{\text{rms}} = \frac{1}{\sqrt{2}}V_0.$$

We may then write for the average power dissipated by a resistor,

$$P_{\text{ave}} = \frac{1}{2}I_0 V_0 = I_{\text{rms}} V_{\text{rms}} = I^2_{\text{rms}} R. \tag{15.13}$$

This equation further emphasizes why the rms value is chosen in discussion rather than peak values. Both equations for average power are correct for **Equation 15.13**, but the rms values in the formula give a cleaner representation, so the extra factor of 1/2 is not necessary.

Alternating voltages and currents are usually described in terms of their rms values. For example, the 110 V from a household outlet is an rms value. The amplitude of this source is $110\sqrt{2}\text{ V} = 156\text{ V}.$ Because most ac meters are calibrated in terms of rms values, a typical ac voltmeter placed across a household outlet will read 110 V.

For a capacitor and an inductor, $\phi = \pi/2$ and $-\pi/2$ rad, respectively. Since $\cos\pi/2 = \cos(-\pi/2) = 0,$ we find from **Equation 15.12** that the average power dissipated by either of these elements is $P_{\text{ave}} = 0.$ Capacitors and inductors absorb energy from the circuit during one half-cycle and then discharge it back to the circuit during the other half-cycle. This behavior is illustrated in the plots of **Figure 15.16**, (b) and (c), which show *p*(t) oscillating sinusoidally about zero.

The phase angle for an ac generator may have any value. If $\cos\phi > 0,$ the generator produces power; if $\cos\phi < 0,$ it absorbs power. In terms of rms values, the average power of an ac generator is written as

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}} \cos\phi.$$

For the generator in an *RLC* circuit,

$$\tan\phi = \frac{X_L - X_C}{R}$$

and

$$\cos\phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{Z}.$$

Hence the average power of the generator is

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}} \cos\phi = \frac{V_{\text{rms}}}{Z} V_{\text{rms}} \frac{R}{Z} = \frac{V_{\text{rms}}^2 R}{Z^2}. \tag{15.14}$$

This can also be written as

$$P_{\text{ave}} = I_{\text{rms}}^2 R,$$

which designates that the power produced by the generator is dissipated in the resistor. As we can see, Ohm's law for the rms ac is found by dividing the rms voltage by the impedance.

Example 15.3

Power Output of a Generator

An ac generator whose emf is given by

$$v(t) = (4.00\text{ V})\sin\left[\left(1.00 \times 10^4\text{ rad/s}\right)t\right]$$

is connected to an *RLC* circuit for which $L = 2.00 \times 10^{-3}\text{ H}$, $C = 4.00 \times 10^{-6}\text{ F}$, and $R = 5.00\,\Omega$. (a) What is the rms voltage across the generator? (b) What is the impedance of the circuit? (c) What is the average power output of the generator?

Strategy

The rms voltage is the amplitude of the voltage times $1/\sqrt{2}$. The impedance of the circuit involves the resistance and the reactances of the capacitor and the inductor. The average power is calculated by **Equation 15.14**, or more specifically, the last part of the equation, because we have the impedance of the circuit *Z*, the rms voltage V_{rms}, and the resistance *R*.

Solution

a. Since $V_0 = 4.00\text{ V},$ the rms voltage across the generator is

$$V_{\text{rms}} = \frac{1}{\sqrt{2}}(4.00\text{ V}) = 2.83\text{ V}.$$

b. The impedance of the circuit is

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \left\{ (5.00\,\Omega)^2 + \left[\left(1.00 \times 10^4 \text{ rad/s}\right)\left(2.00 \times 10^{-3} \text{ H}\right) - \frac{1}{\left(1.00 \times 10^4 \text{ rad/s}\right)\left(4.00 \times 10^{-6} \text{ F}\right)} \right]^2 \right\}^{1/2} \\ &= 7.07\,\Omega. \end{aligned}$$

c. From **Equation 15.14**, the average power transferred to the circuit is

$$P_{\text{ave}} = \frac{V_{\text{rms}}^2 R}{Z^2} = \frac{(2.83 \text{ V})^2 (5.00\,\Omega)}{(7.07\,\Omega)^2} = 0.801 \text{ W}.$$

Significance

If the resistance is much larger than the reactance of the capacitor or inductor, the average power is a dc circuit equation of $P = V^2/R$, where V replaces the rms voltage.

15.4 Check Your Understanding An ac voltmeter attached across the terminals of a 45-Hz ac generator reads 7.07 V. Write an expression for the emf of the generator.

15.5 Check Your Understanding Show that the rms voltages across a resistor, a capacitor, and an inductor in an ac circuit where the rms current is I_{rms} are given by $I_{\text{rms}}R$, $I_{\text{rms}}X_C$, and $I_{\text{rms}}X_L$, respectively. Determine these values for the components of the *RLC* circuit of **Equation 15.12**.

15.5 | Resonance in an AC Circuit

Learning Objectives

By the end of the section, you will be able to:

- Determine the peak ac resonant angular frequency for a RLC circuit
- Explain the width of the average power versus angular frequency curve and its significance using terms like bandwidth and quality factor

In the *RLC* series circuit of **Figure 15.11**, the current amplitude is, from **Equation 15.10**,

$$I_0 = \frac{V_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}. \tag{15.15}$$

If we can vary the frequency of the ac generator while keeping the amplitude of its output voltage constant, then the current changes accordingly. A plot of I_0 versus ω is shown in **Figure 15.17**.

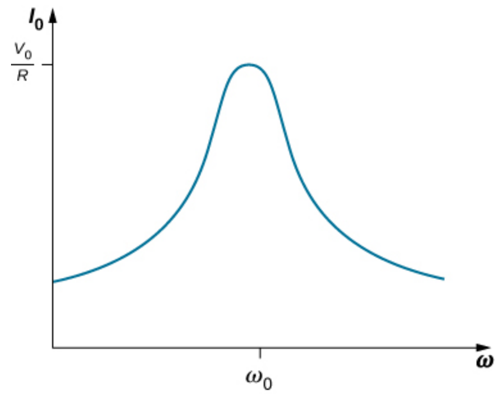


Figure 15.17 At an *RLC* circuit's resonant frequency, $\omega_0 = \sqrt{1/LC}$, the current amplitude is at its maximum value.

In **Oscillations (http://cnx.org/content/m58360/latest/)** , we encountered a similar graph where the amplitude of a damped harmonic oscillator was plotted against the angular frequency of a sinusoidal driving force (see **Forced Oscillations (http://cnx.org/content/m58366/latest/#CNX_UPhysics_15_07_ForcDmpAmp)**). This similarity is more than just a coincidence, as shown earlier by the application of Kirchhoff's loop rule to the circuit of **Figure 15.11**. This yields

$$L\frac{di}{dt} + iR + \frac{q}{C} = V_0 \sin \omega t, \tag{15.16}$$

or

$$L\frac{d^2 q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = V_0 \sin \omega t,$$

where we substituted $dq(\mathrm{t})/dt$ for $i(\mathrm{t})$. A comparison of **Equation 15.16** and, from **Oscillations (http://cnx.org/content/m58360/latest/)** , **Damped Oscillations (http://cnx.org/content/m58365/latest/#fs-id1167131231570)** for damped harmonic motion clearly demonstrates that the driven *RLC* series circuit is the electrical analog of the driven damped harmonic oscillator.

The **resonant frequency** f_0 of the *RLC* circuit is the frequency at which the amplitude of the current is a maximum and the circuit would oscillate if not driven by a voltage source. By inspection, this corresponds to the angular frequency $\omega_0 = 2\pi f_0$ at which the impedance *Z* in **Equation 15.15** is a minimum, or when

$$\omega_0 L = \frac{1}{\omega_0 C}$$

and

$$\omega_0 = \sqrt{\frac{1}{LC}}. \tag{15.17}$$

This is the resonant angular frequency of the circuit. Substituting ω_0 into **Equation 15.9**, **Equation 15.10**, and **Equation 15.11**, we find that at resonance,

$$\phi = \tan^{-1}(0) = 0, \quad I_0 = V_0/R, \quad \text{and} \quad Z = R.$$

Therefore, at resonance, an *RLC* circuit is purely resistive, with the applied emf and current in phase.

What happens to the power at resonance? **Equation 15.14** tells us how the average power transferred from an ac generator to the *RLC* combination varies with frequency. In addition, P_{ave} reaches a maximum when *Z*, which depends on the frequency, is a minimum, that is, when $X_L = X_C$ and $Z = R$. Thus, at resonance, the average power output of the source in an *RLC* series circuit is a maximum. From **Equation 15.14**, this maximum is V_{rms}^2/R.

Figure 15.18 is a typical plot of P_{ave} versus ω in the region of maximum power output. The **bandwidth** $\Delta\omega$ of the resonance peak is defined as the range of angular frequencies ω over which the average power P_{ave} is greater than one-half the maximum value of P_{ave}. The sharpness of the peak is described by a dimensionless quantity known as the **quality factor** *Q* of the circuit. By definition,

$$Q = \frac{\omega_0}{\Delta\omega}, \tag{15.18}$$

where ω_0 is the resonant angular frequency. A high *Q* indicates a sharp resonance peak. We can give *Q* in terms of the circuit parameters as

$$Q = \frac{\omega_0 L}{R}. \tag{15.19}$$

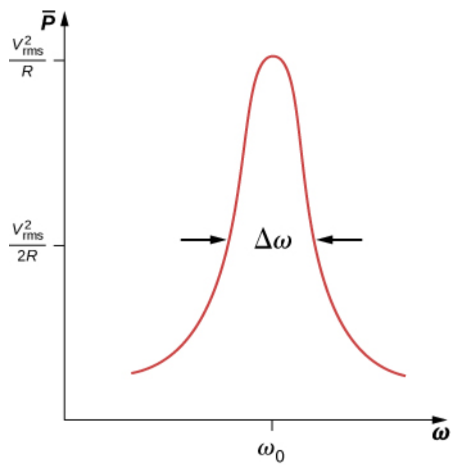


Figure 15.18 Like the current, the average power transferred from an ac generator to an *RLC* circuit peaks at the resonant frequency.

Resonant circuits are commonly used to pass or reject selected frequency ranges. This is done by adjusting the value of one of the elements and hence “tuning” the circuit to a particular resonant frequency. For example, in radios, the receiver is tuned to the desired station by adjusting the resonant frequency of its circuitry to match the frequency of the station. If the tuning circuit has a high *Q*, it will have a small bandwidth, so signals from other stations at frequencies even slightly different from the resonant frequency encounter a high impedance and are not passed by the circuit. Cell phones work in a similar fashion, communicating with signals of around 1 GHz that are tuned by an inductor-capacitor circuit. One of the most common applications of capacitors is their use in ac-timing circuits, based on attaining a resonant frequency. A metal detector also uses a shift in resonance frequency in detecting metals (**Figure 15.19**).

Figure 15.19 When a metal detector comes near a piece of metal, the self-inductance of one of its coils changes. This causes a shift in the resonant frequency of a circuit containing the coil. That shift is detected by the circuitry and transmitted to the diver by means of the headphones. (credit: modification of work by Eric Lippmann, U.S. Navy)

Example 15.4

Resonance in an *RLC* Series Circuit

(a) What is the resonant frequency of the circuit of **Example 15.1**? (b) If the ac generator is set to this frequency without changing the amplitude of the output voltage, what is the amplitude of the current?

Strategy

The resonant frequency for a *RLC* circuit is calculated from **Equation 15.17**, which comes from a balance between the reactances of the capacitor and the inductor. Since the circuit is at resonance, the impedance is equal to the resistor. Then, the peak current is calculated by the voltage divided by the resistance.

Solution

a. The resonant frequency is found from **Equation 15.17**:

$$\begin{aligned} f_0 &= \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{(3.00 \times 10^{-3}\text{ H})(8.00 \times 10^{-4}\text{ F})}} \\ &= 1.03 \times 10^2\text{ Hz}. \end{aligned}$$

b. At resonance, the impedance of the circuit is purely resistive, and the current amplitude is

$$I_0 = \frac{0.100\text{ V}}{4.00\,\Omega} = 2.50 \times 10^{-2}\text{ A}.$$

Significance

If the circuit were not set to the resonant frequency, we would need the impedance of the entire circuit to calculate the current.

Example 15.5

Power Transfer in an *RLC* Series Circuit at Resonance

(a) What is the resonant angular frequency of an *RLC* circuit with $R = 0.200\,\Omega$, $L = 4.00 \times 10^{-3}$ H, and $C = 2.00 \times 10^{-6}$ F? (b) If an ac source of constant amplitude 4.00 V is set to this frequency, what is the average power transferred to the circuit? (c) Determine Q and the bandwidth of this circuit.

Strategy

The resonant angular frequency is calculated from **Equation 15.17**. The average power is calculated from the rms voltage and the resistance in the circuit. The quality factor is calculated from **Equation 15.19** and by knowing the resonant frequency. The bandwidth is calculated from **Equation 15.18** and by knowing the quality factor.

Solution

a. The resonant angular frequency is

$$\begin{aligned}\omega_0 &= \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(4.00 \times 10^{-3}\text{ H})(2.00 \times 10^{-6}\text{ F})}} \\ &= 1.12 \times 10^4 \text{ rad/s.}\end{aligned}$$

b. At this frequency, the average power transferred to the circuit is a maximum. It is

$$P_{\text{ave}} = \frac{V_{\text{rms}}^2}{R} = \frac{[(1/\sqrt{2})(4.00\text{ V})]^2}{0.200\,\Omega} = 40.0\text{ W}.$$

c. The quality factor of the circuit is

$$Q = \frac{\omega_0 L}{R} = \frac{(1.12 \times 10^4\text{ rad/s})(4.00 \times 10^{-3}\text{ H})}{0.200\,\Omega} = 224.$$

We then find for the bandwidth

$$\Delta\omega = \frac{\omega_0}{Q} = \frac{1.12 \times 10^4\text{ rad/s}}{224} = 50.0\text{ rad/s}.$$

Significance

If a narrower bandwidth is desired, a lower resistance or higher inductance would help. However, a lower resistance increases the power transferred to the circuit, which may not be desirable, depending on the maximum power that could possibly be transferred.

15.6 Check Your Understanding In the circuit of **Figure 15.11**, $L = 2.0 \times 10^{-3}$ H, $C = 5.0 \times 10^{-4}$ F, and $R = 40\,\Omega$. (a) What is the resonant frequency? (b) What is the impedance of the circuit at resonance? (c) If the voltage amplitude is 10 V, what is $i(t)$ at resonance? (d) The frequency of the AC generator is now changed to 200 Hz. Calculate the phase difference between the current and the emf of the generator.

15.7 Check Your Understanding What happens to the resonant frequency of an *RLC* series circuit when the following quantities are increased by a factor of 4: (a) the capacitance, (b) the self-inductance, and (c) the resistance?

15.8 Check Your Understanding The resonant angular frequency of an *RLC* series circuit is 4.0×10^2 rad/s. An ac source operating at this frequency transfers an average power of 2.0×10^{-2} W to the circuit. The resistance of the circuit is $0.50\,\Omega$. Write an expression for the emf of the source.

15.6 | Transformers

Learning Objectives

By the end of the section, you will be able to:

- Explain why power plants transmit electricity at high voltages and low currents and how they do this
- Develop relationships among current, voltage, and the number of windings in step-up and step-down transformers

Although ac electric power is produced at relatively low voltages, it is sent through transmission lines at very high voltages (as high as 500 kV). The same power can be transmitted at different voltages because power is the product $I_{\text{rms}} V_{\text{rms}}$. (For simplicity, we ignore the phase factor $\cos\phi$.) A particular power requirement can therefore be met with a low voltage and a high current or with a high voltage and a low current. The advantage of the high-voltage/low-current choice is that it results in lower $I_{\text{rms}}^2 R$ ohmic losses in the transmission lines, which can be significant in lines that are many kilometers long (**Figure 15.20**).

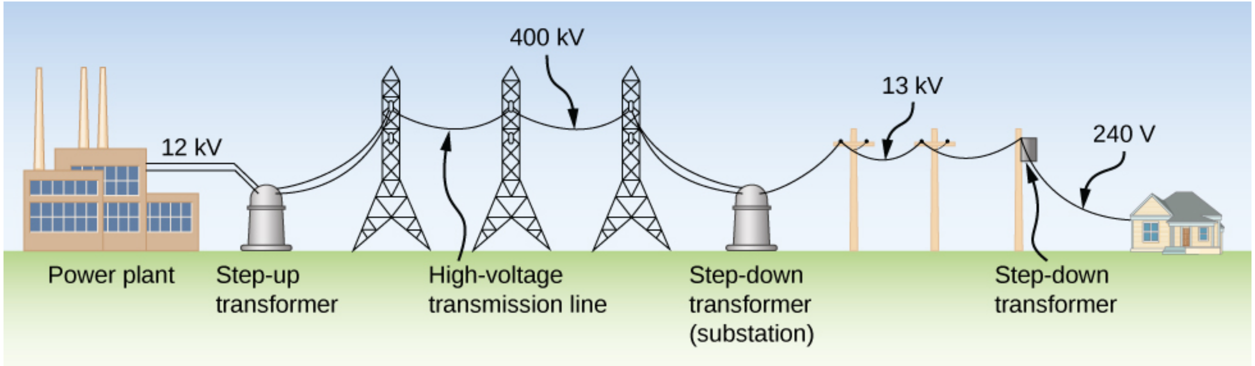


Figure 15.20 The rms voltage from a power plant eventually needs to be stepped down from 12 kV to 240 V so that it can be safely introduced into a home. A high-voltage transmission line allows a low current to be transmitted via a substation over long distances.

Typically, the alternating emfs produced at power plants are “stepped up” to very high voltages before being transmitted through power lines; then, they must be “stepped down” to relatively safe values (110 or 220 V rms) before they are introduced into homes. The device that transforms voltages from one value to another using induction is the **transformer** (**Figure 15.21**).

Figure 15.21 Transformers are used to step down the high voltages in transmission lines to the 110 to 220 V used in homes. (credit: modification of work by “Fortyseven”/Flickr)

As **Figure 15.22** illustrates, a transformer basically consists of two separated coils, or windings, wrapped around a soft iron core. The primary winding has N_{P} loops, or turns, and is connected to an alternating voltage $v_{\text{P}}(t)$. The secondary winding has N_{S} turns and is connected to a load resistor R_{S}. We assume the ideal case for which all magnetic field lines are confined to the core so that the same magnetic flux permeates each turn of both the primary and the secondary windings. We also neglect energy losses to magnetic hysteresis, to ohmic heating in the windings, and to ohmic heating of the induced eddy currents in the core. A good transformer can have losses as low as 1% of the transmitted power, so this is not a bad assumption.

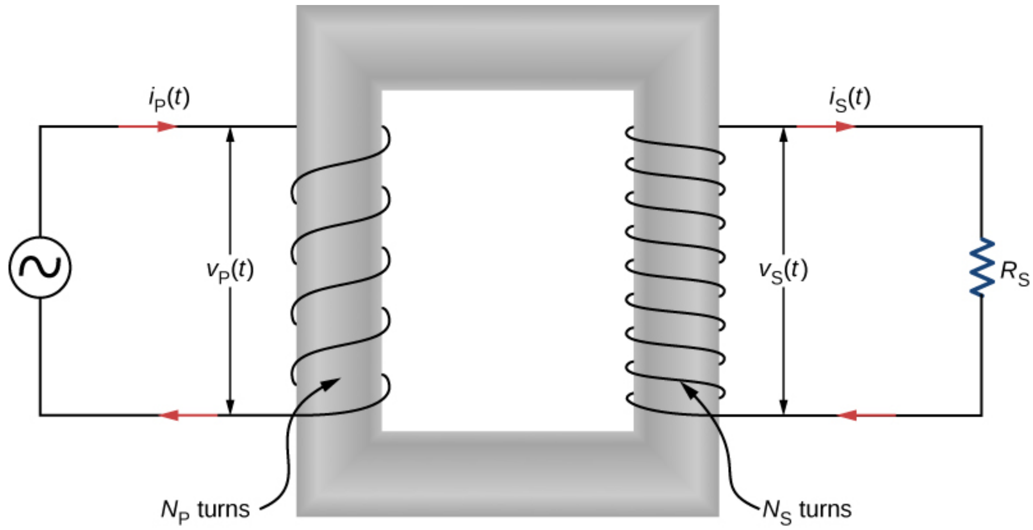


Figure 15.22 A step-up transformer (more turns in the secondary winding than in the primary winding). The two windings are wrapped around a soft iron core.

To analyze the transformer circuit, we first consider the primary winding. The input voltage $v_P(t)$ is equal to the potential difference induced across the primary winding. From Faraday's law, the induced potential difference is $-N_P(d\Phi/dt)$, where Φ is the flux through one turn of the primary winding. Thus,

$$v_P(t) = -N_P \frac{d\Phi}{dt}.$$

Similarly, the output voltage $v_S(t)$ delivered to the load resistor must equal the potential difference induced across the secondary winding. Since the transformer is ideal, the flux through every turn of the secondary winding is also Φ, and

$$v_S(t) = -N_S \frac{d\Phi}{dt}.$$

Combining the last two equations, we have

$$v_S(t) = \frac{N_S}{N_P} v_P(t). \tag{15.20}$$

Hence, with appropriate values for N_S and N_P, the input voltage $v_P(t)$ may be "stepped up" $(N_S > N_P)$ or "stepped down" ($N_S < N_P$) to $v_S(t)$, the output voltage. This is often abbreviated as the **transformer equation**,

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}, \tag{15.21}$$

which shows that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of turns in their windings. For a **step-up transformer**, which increases voltage and decreases current, this ratio is greater than one; for a **step-down transformer**, which decreases voltage and increases current, this ratio is less than one.

From the law of energy conservation, the power introduced at any instant by $v_P(t)$ to the primary winding must be equal to the power dissipated in the resistor of the secondary circuit; thus,

$$i_P(t) v_P(t) = i_S(t) v_S(t).$$

When combined with **Equation 15.20**, this gives

$$i_S(t) = \frac{N_P}{N_S} i_P(t). \tag{15.22}$$

If the voltage is stepped up, the current is stepped down, and vice versa.

Finally, we can use $i_S(t) = v_S(t)/R_S$, along with Equation 15.20 and Equation 15.22, to obtain

$$v_P(t) = i_P\left[\left(\frac{N_P}{N_S}\right)^2 R_S\right],$$

which tells us that the input voltage $v_P(t)$ "sees" not a resistance R_S but rather a resistance

$$R_P = \left(\frac{N_P}{N_S}\right)^2 R_S.$$

Our analysis has been based on instantaneous values of voltage and current. However, the resulting equations are not limited to instantaneous values; they hold also for maximum and rms values.

Example 15.6

A Step-Down Transformer

A transformer on a utility pole steps the rms voltage down from 12 kV to 240 V. (a) What is the ratio of the number of secondary turns to the number of primary turns? (b) If the input current to the transformer is 2.0 A, what is the output current? (c) Determine the power loss in the transmission line if the total resistance of the transmission line is $200\,\Omega$. (d) What would the power loss have been if the transmission line was at 240 V the entire length of the line, rather than providing voltage at 12 kV? What does this say about transmission lines?

Strategy

The number of turns related to the voltages is found from Equation 15.20. The output current is calculated using Equation 15.22.

Solution

a. Using Equation 15.20 with rms values V_P and V_S, we have

$$\frac{N_S}{N_P} = \frac{240\text{ V}}{12 \times 10^3\text{ V}} = \frac{1}{50},$$

so the primary winding has 50 times the number of turns in the secondary winding.

b. From Equation 15.22, the output rms current I_S is found using the transformer equation with current

$$I_S = \frac{N_P}{N_S} I_P \tag{15.23}$$

such that

$$I_S = \frac{N_P}{N_S} I_P = (50)(2.0\text{ A}) = 100\text{ A}.$$

c. The power loss in the transmission line is calculated to be

$$P_{\text{loss}} = I_P^2 R = (2.0\text{ A})^2 (200\,\Omega) = 800\text{ W}.$$

d. If there were no transformer, the power would have to be sent at 240 V to work for these houses, and the power loss would be

$$P_{\text{loss}} = I_S^2 R = (100\ \text{A})^2 (200\ \Omega) = 2 \times 10^6\ \text{W}.$$

Therefore, when power needs to be transmitted, we want to avoid power loss. Thus, lines are sent with high voltages and low currents and adjusted with a transformer before power is sent into homes.

Significance

This application of a step-down transformer allows a home that uses 240-V outlets to have 100 A available to draw upon. This can power many devices in the home.

15.9 Check Your Understanding A transformer steps the line voltage down from 110 to 9.0 V so that a current of 0.50 A can be delivered to a doorbell. (a) What is the ratio of the number of turns in the primary and secondary windings? (b) What is the current in the primary winding? (c) What is the resistance seen by the 110-V source?

CHAPTER 15 REVIEW

KEY TERMS

ac current current that fluctuates sinusoidally with time at a fixed frequency

ac voltage voltage that fluctuates sinusoidally with time at a fixed frequency

alternating current (ac) flow of electric charge that periodically reverses direction

average power time average of the instantaneous power over one cycle

bandwidth range of angular frequencies over which the average power is greater than one-half the maximum value of the average power

capacitive reactance opposition of a capacitor to a change in current

direct current (dc) flow of electric charge in only one direction

impedance ac analog to resistance in a dc circuit, which measures the combined effect of resistance, capacitive reactance, and inductive reactance

inductive reactance opposition of an inductor to a change in current

phase angle amount by which the voltage and current are out of phase with each other in a circuit

power factor amount by which the power delivered in the circuit is less than the theoretical maximum of the circuit due to voltage and current being out of phase

quality factor dimensionless quantity that describes the sharpness of the peak of the bandwidth; a high quality factor is a sharp or narrow resonance peak

resonant frequency frequency at which the amplitude of the current is a maximum and the circuit would oscillate if not driven by a voltage source

rms current root mean square of the current

rms voltage root mean square of the voltage

step-down transformer transformer that decreases voltage and increases current

step-up transformer transformer that increases voltage and decreases current

transformer device that transforms voltages from one value to another using induction

transformer equation equation showing that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of turns in their windings

KEY EQUATIONS

AC voltage	$v = V_0 \sin \omega t$
AC current	$i = I_0 \sin \omega t$
capacitive reactance	$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C$
rms voltage	$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$
rms current	$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$
inductive reactance	$\frac{V_0}{I_0} = \omega L = X_L$

Phase angle of an ac circuit	$\phi = \tan^{-1}\dfrac{X_L - X_C}{R}$
AC version of Ohm's law	$I_0 = \dfrac{V_0}{Z}$
Impedance of an ac circuit	$Z = \sqrt{R^2 + (X_L - X_C)^2}$
Average power associated with a circuit element	$P_{\text{ave}} = \frac{1}{2}I_0 V_0 \cos\phi$
Average power dissipated by a resistor	$P_{\text{ave}} = \frac{1}{2}I_0 V_0 = I_{\text{rms}} V_{\text{rms}} = I_{\text{rms}}^2 R$
Resonant angular frequency of a circuit	$\omega_0 = \sqrt{\dfrac{1}{LC}}$
Quality factor of a circuit	$Q = \dfrac{\omega_0}{\Delta\omega}$
Quality factor of a circuit in terms of the circuit parameters	$Q = \dfrac{\omega_0 L}{R}$
Transformer equation with voltage	$\dfrac{V_{\text{S}}}{V_{\text{P}}} = \dfrac{N_{\text{S}}}{N_{\text{P}}}$
Transformer equation with current	$I_{\text{S}} = \dfrac{N_{\text{P}}}{N_{\text{S}}} I_{\text{P}}$

SUMMARY

15.1 AC Sources

- Direct current (dc) refers to systems in which the source voltage is constant.
- Alternating current (ac) refers to systems in which the source voltage varies periodically, particularly sinusoidally.
- The voltage source of an ac system puts out a voltage that is calculated from the time, the peak voltage, and the angular frequency.
- In a simple circuit, the current is found by dividing the voltage by the resistance. An ac current is calculated using the peak current (determined by dividing the peak voltage by the resistance), the angular frequency, and the time.

15.2 Simple AC Circuits

- For resistors, the current through and the voltage across are in phase.
- For capacitors, we find that when a sinusoidal voltage is applied to a capacitor, the voltage follows the current by one-fourth of a cycle. Since a capacitor can stop current when fully charged, it limits current and offers another form of ac resistance, called capacitive reactance, which has units of ohms.
- For inductors in ac circuits, we find that when a sinusoidal voltage is applied to an inductor, the voltage leads the current by one-fourth of a cycle.
- The opposition of an inductor to a change in current is expressed as a type of ac reactance. This inductive reactance, which has units of ohms, varies with the frequency of the ac source.

15.3 RLC Series Circuits with AC

- An *RLC* series circuit is a resistor, capacitor, and inductor series combination across an ac source.
- The same current flows through each element of an *RLC* series circuit at all points in time.
- The counterpart of resistance in a dc circuit is impedance, which measures the combined effect of resistors, capacitors, and inductors. The maximum current is defined by the ac version of Ohm's law.

- Impedance has units of ohms and is found using the resistance, the capacitive reactance, and the inductive reactance.

15.4 Power in an AC Circuit

- The average ac power is found by multiplying the rms values of current and voltage.
- Ohm’s law for the rms ac is found by dividing the rms voltage by the impedance.
- In an ac circuit, there is a phase angle between the source voltage and the current, which can be found by dividing the resistance by the impedance.
- The average power delivered to an *RLC* circuit is affected by the phase angle.
- The power factor ranges from –1 to 1.

15.5 Resonance in an AC Circuit

- At the resonant frequency, inductive reactance equals capacitive reactance.
- The average power versus angular frequency plot for a *RLC* circuit has a peak located at the resonant frequency; the sharpness or width of the peak is known as the bandwidth.
- The bandwidth is related to a dimensionless quantity called the quality factor. A high quality factor value is a sharp or narrow peak.

15.6 Transformers

- Power plants transmit high voltages at low currents to achieve lower ohmic losses in their many kilometers of transmission lines.
- Transformers use induction to transform voltages from one value to another.
- For a transformer, the voltages across the primary and secondary coils, or windings, are related by the transformer equation.
- The currents in the primary and secondary windings are related by the number of primary and secondary loops, or turns, in the windings of the transformer.
- A step-up transformer increases voltage and decreases current, whereas a step-down transformer decreases voltage and increases current.

CONCEPTUAL QUESTIONS

15.1 AC Sources

1. What is the relationship between frequency and angular frequency?

15.2 Simple AC Circuits

2. Explain why at high frequencies a capacitor acts as an ac short, whereas an inductor acts as an open circuit.

15.3 RLC Series Circuits with AC

3. In an *RLC* series circuit, can the voltage measured across the capacitor be greater than the voltage of the source? Answer the same question for the voltage across the inductor.

15.4 Power in an AC Circuit

4. For what value of the phase angle ϕ between the voltage output of an ac source and the current is the average power output of the source a maximum?

5. Discuss the differences between average power and instantaneous power.

6. The average ac current delivered to a circuit is zero. Despite this, power is dissipated in the circuit. Explain.

7. Can the instantaneous power output of an ac source ever be negative? Can the average power output be negative?

8. The power rating of a resistor used in ac circuits refers to the maximum average power dissipated in the resistor. How does this compare with the maximum instantaneous power dissipated in the resistor?

15.6 Transformers

9. Why do transmission lines operate at very high voltages while household circuits operate at fairly small voltages?

10. How can you distinguish the primary winding from the secondary winding in a step-up transformer?

11. Battery packs in some electronic devices are charged using an adapter connected to a wall socket. Speculate as to the purpose of the adapter.

12. Will a transformer work if the input is a dc voltage?

13. Why are the primary and secondary coils of a transformer wrapped around the same closed loop of iron?

PROBLEMS

15.1 AC Sources

14. Write an expression for the output voltage of an ac source that has an amplitude of 12 V and a frequency of 200 Hz.

15.2 Simple AC Circuits

15. Calculate the reactance of a $5.0\text{-}\mu\text{F}$ capacitor at (a) 60 Hz, (b) 600 Hz, and (c) 6000 Hz.

16. What is the capacitance of a capacitor whose reactance is $10\,\Omega$ at 60 Hz?

17. Calculate the reactance of a 5.0-mH inductor at (a) 60 Hz, (b) 600 Hz, and (c) 6000 Hz.

18. What is the self-inductance of a coil whose reactance is $10\,\Omega$ at 60 Hz?

19. At what frequency is the reactance of a $20\text{-}\mu\text{F}$ capacitor equal to that of a 10-mH inductor?

20. At 1000 Hz, the reactance of a 5.0-mH inductor is equal to the reactance of a particular capacitor. What is the capacitance of the capacitor?

21. A $50\text{-}\Omega$ resistor is connected across the emf $v(t) = (160\text{ V})\sin(120\pi t)$. Write an expression for the current through the resistor.

22. A $25\text{-}\mu\text{F}$ capacitor is connected to an emf given by $v(t) = (160\text{ V})\sin(120\pi t)$. (a) What is the reactance of the capacitor? (b) Write an expression for the current output of the source.

23. A 100-mH inductor is connected across the emf of the preceding problem. (a) What is the reactance of the inductor? (b) Write an expression for the current through the inductor.

15.3 RLC Series Circuits with AC

24. What is the impedance of a series combination of a $50\text{-}\Omega$ resistor, a $5.0\text{-}\mu\text{F}$ capacitor, and a $10\text{-}\mu\text{F}$ capacitor at a frequency of 2.0 kHz?

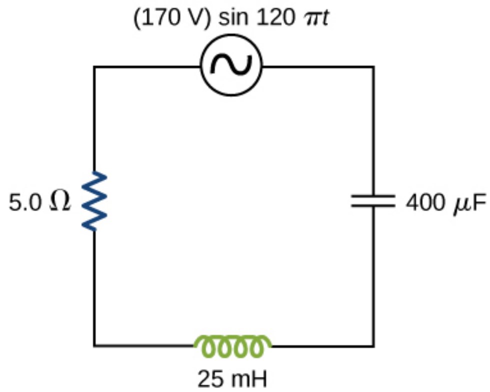
25. A resistor and capacitor are connected in series across an ac generator. The emf of the generator is given by $v(t) = V_0 \cos \omega t$, where $V_0 = 120\text{ V}$, $\omega = 120\pi\text{ rad/s}$, $R = 400\,\Omega$, and $C = 4.0\mu\text{F}$. (a) What is the impedance of the circuit? (b) What is the amplitude of the current through the resistor? (c) Write an expression for the current through the resistor. (d) Write expressions representing the voltages across the resistor and across the capacitor.

26. A resistor and inductor are connected in series across an ac generator. The emf of the generator is given by $v(t) = V_0 \cos \omega t$, where $V_0 = 120\text{ V}$ and $\omega = 120\pi\text{ rad/s}$; also, $R = 400\,\Omega$ and $L = 1.5\text{ H}$. (a) What is the impedance of the circuit? (b) What is the amplitude of the current through the resistor? (c) Write an expression for the current through the resistor. (d) Write expressions representing the voltages across the resistor and across the inductor.

27. In an *RLC* series circuit, the voltage amplitude and frequency of the source are 100 V and 500 Hz, respectively, an $R = 500\,\Omega$, $L = 0.20\text{ H}$, and $C = 2.0\mu\text{F}$. (a) What is the impedance of the circuit? (b) What is the amplitude of the current from the source? (c) If the emf of the source is given by $v(t) = (100\text{ V})\sin 1000\pi t$, how does the current vary with time? (d) Repeat the calculations with *C* changed to $0.20\mu\text{F}$.

28. An *RLC* series circuit with $R = 600\,\Omega$, $L = 30\text{ mH}$, and $C = 0.050\mu\text{F}$ is driven by an ac source whose frequency and voltage amplitude are 500 Hz and 50 V, respectively. (a) What is the impedance of the circuit? (b) What is the amplitude of the current in the circuit? (c) What is the phase angle between the emf of the source and the current?

29. For the circuit shown below, what are (a) the total impedance and (b) the phase angle between the current and the emf? (c) Write an expression for $i(t)$.



15.4 Power in an AC Circuit

30. The emf of an ac source is given by $v(t) = V_0 \sin \omega t$, where $V_0 = 100\text{ V}$ and $\omega = 200\pi\text{ rad/s}$. Calculate the average power output of the source if it is connected across (a) a 20-μF capacitor, (b) a 20-mH inductor, and (c) a 50-Ω resistor.

31. Calculate the rms currents for an ac source is given by $v(t) = V_0 \sin \omega t$, where $V_0 = 100\text{ V}$ and $\omega = 200\pi\text{ rad/s}$ when connected across (a) a 20-μF capacitor, (b) a 20-mH inductor, and (c) a 50-Ω resistor.

32. A 40-mH inductor is connected to a 60-Hz AC source whose voltage amplitude is 50 V. If an AC voltmeter is placed across the inductor, what does it read?

33. For an *RLC* series circuit, the voltage amplitude and frequency of the source are 100 V and 500 Hz, respectively; $R = 500\,\Omega$; and $L = 0.20\text{ H}$. Find the average power dissipated in the resistor for the following values for the capacitance: (a) $C = 2.0\mu\text{F}$ and (b) $C = 0.20\,\mu\text{F}$.

34. An ac source of voltage amplitude 10 V delivers electric energy at a rate of 0.80 W when its current output is 2.5 A. What is the phase angle ϕ between the emf and the current?

35. An *RLC* series circuit has an impedance of $60\,\Omega$ and a power factor of 0.50, with the voltage lagging the current. (a) Should a capacitor or an inductor be placed in series with the elements to raise the power factor of the circuit? (b) What is the value of the capacitance or self-inductance that will raise the power factor to unity?

15.5 Resonance in an AC Circuit

36. (a) Calculate the resonant angular frequency of an *RLC* series circuit for which $R = 20\,\Omega$, $L = 75\text{ mH}$, and $C = 4.0\mu\text{F}$. (b) If *R* is changed to $300\,\Omega$, what happens to the resonant angular frequency?

37. The resonant frequency of an *RLC* series circuit is $2.0 \times 10^3\text{ Hz}$. If the self-inductance in the circuit is 5.0 mH, what is the capacitance in the circuit?

38. (a) What is the resonant frequency of an *RLC* series circuit with $R = 20\,\Omega$, $L = 2.0\text{ mH}$, and $C = 4.0\mu\text{F}$? (b) What is the impedance of the circuit at resonance?

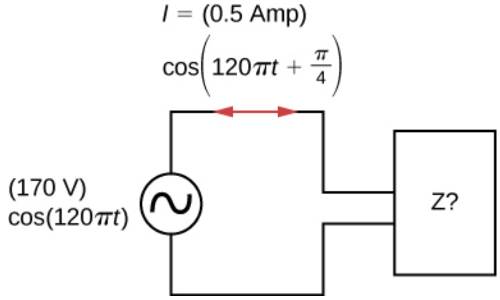
39. For an *RLC* series circuit, $R = 100\,\Omega$, $L = 150\text{ mH}$, and $C = 0.25\mu\text{F}$. (a) If an ac source of variable frequency is connected to the circuit, at what frequency is maximum power dissipated in the resistor? (b) What is the quality factor of the circuit?

40. An ac source of voltage amplitude 100 V and variable frequency *f* drives an *RLC* series circuit with $R = 10\,\Omega$, $L = 2.0\text{ mH}$, and $C = 25\mu\text{F}$. (a) Plot the current through the resistor as a function of the frequency *f*. (b) Use the plot to determine the resonant frequency of the circuit.

41. (a) What is the resonant frequency of a resistor, capacitor, and inductor connected in series if $R = 100\,\Omega$, $L = 2.0\text{ H}$, and $C = 5.0\mu\text{F}$? (b) If this combination is connected to a 100-V source operating at the resonant frequency, what is the power output of the source? (c) What is the *Q* of the circuit? (d) What is the bandwidth of the circuit?

42. Suppose a coil has a self-inductance of 20.0 H and a resistance of $200\,\Omega$. What (a) capacitance and (b) resistance must be connected in series with the coil to produce a circuit that has a resonant frequency of 100 Hz and a *Q* of 10?

43. An ac generator is connected to a device whose internal circuits are not known. We only know current and voltage outside the device, as shown below. Based on the information given, what can you infer about the electrical nature of the device and its power usage?



15.6 Transformers

44. A step-up transformer is designed so that the output of its secondary winding is 2000 V (rms) when the primary winding is connected to a 110-V (rms) line voltage. (a) If there are 100 turns in the primary winding, how many turns are there in the secondary winding? (b) If a resistor connected across the secondary winding draws an rms current of 0.75 A, what is the current in the primary winding?

45. A step-up transformer connected to a 110-V line is used to supply a hydrogen-gas discharge tube with 5.0 kV (rms). The tube dissipates 75 W of power. (a) What is the ratio of the number of turns in the secondary winding to the number of turns in the primary winding? (b) What are the rms currents in the primary and secondary windings? (c) What is the effective resistance seen by the 110-V source?

46. An ac source of emf delivers 5.0 mW of power at an rms current of 2.0 mA when it is connected to the primary coil of a transformer. The rms voltage across the secondary coil is 20 V. (a) What are the voltage across the primary coil and the current through the secondary coil? (b) What is the ratio of secondary to primary turns for the transformer?

47. A transformer is used to step down 110 V from a wall socket to 9.0 V for a radio. (a) If the primary winding has 500 turns, how many turns does the secondary winding have? (b) If the radio operates at a current of 500 mA, what is the current through the primary winding?

48. A transformer is used to supply a 12-V model train with power from a 110-V wall plug. The train operates at 50 W of power. (a) What is the rms current in the secondary coil of the transformer? (b) What is the rms current in the primary coil? (c) What is the ratio of the number of primary to secondary turns? (d) What is the resistance of the train? (e) What is the resistance seen by the 110-V source?

ADDITIONAL PROBLEMS

49. The emf of an ac source is given by $v(t) = V_0 \sin \omega t$, where $V_0 = 100\text{ V}$ and $\omega = 200\pi\text{ rad/s}$. Find an expression that represents the output current of the source if it is connected across (a) a $20\text{-}\mu\text{F}$ capacitor, (b) a 20-mH inductor, and (c) a $50\text{-}\Omega$ resistor.

50. A 700-pF capacitor is connected across an ac source with a voltage amplitude of 160 V and a frequency of 20 kHz. (a) Determine the capacitive reactance of the capacitor and the amplitude of the output current of the source. (b) If the frequency is changed to 60 Hz while keeping the voltage amplitude at 160 V, what are the capacitive reactance and the current amplitude?

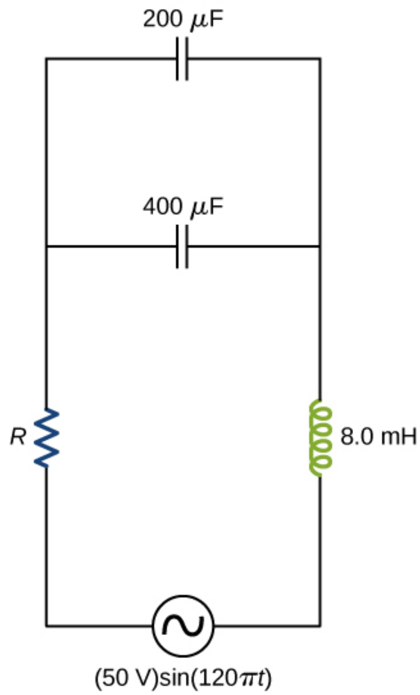
51. A 20-mH inductor is connected across an AC source with a variable frequency and a constant-voltage amplitude of 9.0 V. (a) Determine the reactance of the circuit and the maximum current through the inductor when the frequency is set at 20 kHz. (b) Do the same calculations for a frequency of 60 Hz.

52. A $30\text{-}\mu\text{F}$ capacitor is connected across a 60-Hz ac source whose voltage amplitude is 50 V. (a) What is the maximum charge on the capacitor? (b) What is the maximum current into the capacitor? (c) What is the phase relationship between the capacitor charge and the current in the circuit?

53. A 7.0-mH inductor is connected across a 60-Hz ac source whose voltage amplitude is 50 V. (a) What is the maximum current through the inductor? (b) What is the phase relationship between the current through and the potential difference across the inductor?

54. What is the impedance of an *RLC* series circuit at the resonant frequency?

55. What is the resistance *R* in the circuit shown below if the amplitude of the ac through the inductor is 4.24 A?



56. An ac source of voltage amplitude 100 V and frequency 1.0 kHz drives an *RLC* series circuit with $R = 20\,\Omega$, $L = 4.0\,\text{mH}$, and $C = 50\mu\text{F}$. (a) Determine the rms current through the circuit. (b) What are the rms voltages across the three elements? (c) What is the phase angle between the emf and the current? (d) What is the power output of the source? (e) What is the power dissipated in the resistor?

57. In an *RLC* series circuit, $R = 200\,\Omega$, $L = 1.0\,\text{H}$, $C = 50\mu\text{F}$, $V_0 = 120\,\text{V}$, and $f = 50\,\text{Hz}$. What is the power output of the source?

58. A power plant generator produces 100 A at 15 kV (rms). A transformer is used to step up the transmission line voltage to 150 kV (rms). (a) What is rms current in the transmission line? (b) If the resistance per unit length of the line is $8.6 \times 10^{-8}\ \Omega/\text{m}$, what is the power loss per meter in the line? (c) What would the power loss per meter be if the line voltage were 15 kV (rms)?

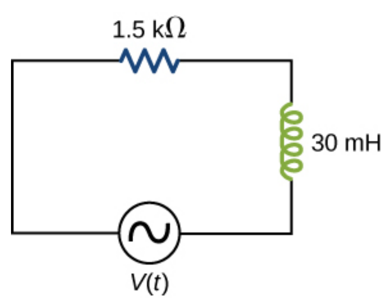
59. Consider a power plant located 25 km outside a town delivering 50 MW of power to the town. The transmission lines are made of aluminum cables with a $7\,\text{cm}^2$ cross-sectional area. Find the loss of power in the transmission lines if it is transmitted at (a) 200 kV (rms) and (b) 120 V (rms).

60. Neon signs require 12-kV for their operation. A transformer is to be used to change the voltage from 220-V (rms) ac to 12-kV (rms) ac. What must the ratio be of turns in the secondary winding to the turns in the primary winding? (b) What is the maximum rms current the neon lamps can draw if the fuse in the primary winding goes off at 0.5 A? (c) How much power is used by the neon sign when it is drawing the maximum current allowed by the fuse in the primary winding?

CHALLENGE PROBLEMS

61. The 335-kV ac electricity from a power transmission line is fed into the primary winding of a transformer. The ratio of the number of turns in the secondary winding to the number in the primary winding is $N_s/N_p = 1000$. (a) What voltage is induced in the secondary winding? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

62. A 1.5-kΩ resistor and 30-mH inductor are connected in series, as shown below, across a 120-V (rms) ac power source oscillating at 60-Hz frequency. (a) Find the current in the circuit. (b) Find the voltage drops across the resistor and inductor. (c) Find the impedance of the circuit. (d) Find the power dissipated in the resistor. (e) Find the power dissipated in the inductor. (f) Find the power produced by the source.

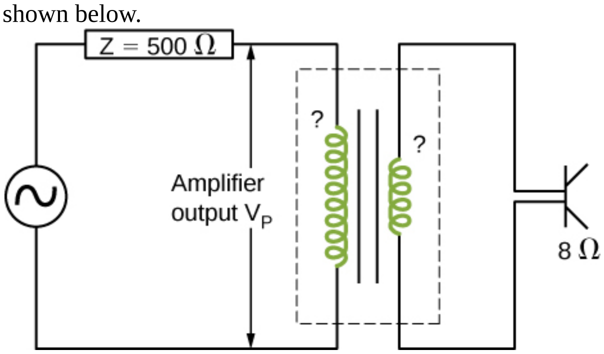


63. A 20-Ω resistor, 50-μF capacitor, and 30-mH inductor are connected in series with an ac source of amplitude 10 V and frequency 125 Hz. (a) What is the impedance of the circuit? (b) What is the amplitude of the current in the circuit? (c) What is the phase constant of the current? Is it leading or lagging the source voltage? (d) Write voltage drops across the resistor, capacitor, and inductor and the source voltage as a function of time. (e) What is the power factor of the circuit? (f) How much energy is used by the resistor in 2.5 s?

64. A $200\text{-}\Omega$ resistor, $150\text{-}\mu\text{F}$ capacitor, and 2.5-H inductor are connected in series with an ac source of amplitude 10 V and variable angular frequency ω. (a) What is the value of the resonance frequency ω_R? (b) What is the amplitude of the current if $\omega = \omega_R$? (c) What is the phase constant of the current when $\omega = \omega_R$? Is it leading or lagging the source voltage, or is it in phase? (d) Write an equation for the voltage drop across the resistor as a function of time when $\omega = \omega_R$. (e) What is the power factor of the circuit when $\omega = \omega_R$? (f) How much energy is used up by the resistor in 2.5 s when $\omega = \omega_R$?

65. Find the reactances of the following capacitors and inductors in ac circuits with the given frequencies in each case: (a) 2-mH inductor with a frequency 60-Hz of the ac circuit; (b) 2-mH inductor with a frequency 600-Hz of the ac circuit; (c) 20-mH inductor with a frequency 6-Hz of the ac circuit; (d) 20-mH inductor with a frequency 60-Hz of the ac circuit; (e) 2-mF capacitor with a frequency 60-Hz of the ac circuit; and (f) 2-mF capacitor with a frequency 600-Hz of the AC circuit.

66. An output impedance of an audio amplifier has an impedance of $500\,\Omega$ and has a mismatch with a low-impedance $8\text{-}\Omega$ loudspeaker. You are asked to insert an appropriate transformer to match the impedances. What turns ratio will you use, and why? Use the simplified circuit shown below.



67. Show that the SI unit for capacitive reactance is the ohm. Show that the SI unit for inductive reactance is also the ohm.

68. A coil with a self-inductance of 16 mH and a resistance of $6.0\,\Omega$ is connected to an ac source whose frequency can be varied. At what frequency will the voltage across the coil lead the current through the coil by $45°$?

69. An *RLC* series circuit consists of a $50\text{-}\Omega$ resistor, a $200\text{-}\mu\text{F}$ capacitor, and a 120-mH inductor whose coil has a resistance of $20\,\Omega$. The source for the circuit has an rms emf of 240 V at a frequency of 60 Hz. Calculate the rms voltages across the (a) resistor, (b) capacitor, and (c) inductor.

70. An *RLC* series circuit consists of a $10\text{-}\Omega$ resistor, an $8.0\text{-}\mu\text{F}$ capacitor, and a 50-mH inductor. A 110-V (rms) source of variable frequency is connected across the combination. What is the power output of the source when its frequency is set to one-half the resonant frequency of the circuit?

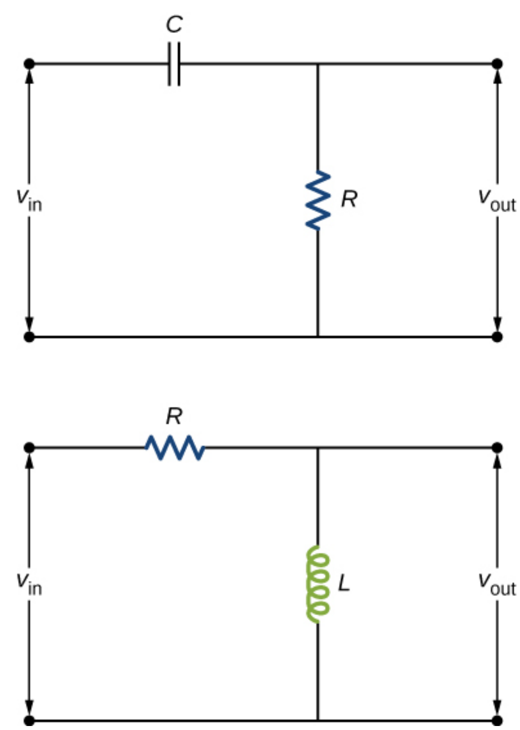
71. Shown below are two circuits that act as crude high-pass filters. The input voltage to the circuits is v_{in}, and the output voltage is v_{out}. (a) Show that for the capacitor circuit,

$$\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{1}{\sqrt{1 + 1/\omega^2 R^2 C^2}},$$

and for the inductor circuit,

$$\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}}.$$

(b) Show that for high frequencies, $v_{\text{out}} \approx v_{\text{in}}$, but for low frequencies, $v_{\text{out}} \approx 0$.



72. The two circuits shown below act as crude low-pass filters. The input voltage to the circuits is v_{in}, and the output voltage is v_{out}. (a) Show that for the capacitor circuit,

$$\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}},$$

and for the inductor circuit,

$$\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}.$$

(b) Show that for low frequencies, $v_{\text{out}} \approx v_{\text{in}}$, but for high frequencies, $v_{\text{out}} \approx 0$.